Baroque Form Generation Practices
A Historical Study

Lydia M. Soo
University of Michigan

Abstract

Adherents to 'blob' design, made possible through digital technologies, have interrogated Baroque architecture in various ways relating to form, process, representation, and affect. In doing so they have developed narratives that are about 'citation': the creation of new insights for the production of architecture today based on a Baroque subject mediated by philosophers and theorists. Alternatively, this paper offers an approach based on 'quotation' in the sense of breaking the artificial continuity from the Baroque to the present. It does so by considering one issue: the form generation practices used by architects of that period. A historical study based on primary sources and hands-on experimentation with tools and methods dating back to the Middle Ages and antiquity, it demonstrates how the Baroque designer achieved built form not blindly or arbitrarily, but based on certain well thought out intentions. With compass and rule in hand, architects drew the geometrical procedures of the biangolo*, quadrature, and the oval, first published in Serlio's 1545 book on geometry. These procedures can be traced in Renaissance designs, but also in the complex curvilinear forms of the Baroque, including Guarini's San Lorenzo. This design and his treatise provide evidence that architects began with a vision. Guarini's involved curvilinear and intersecting forms, ribbed and layered domes, and extremes of light, resulting in visual effects of infinity, weightlessness, and dematerialisation. This vision came out of his study of mathematics and optics, but also direct experience with buildings from a broad range of periods and cultures. Proceeding with knowledge and intent rather than indifference, controlling his tools and methods rather than giving them free rein, the Baroque architect created a vision and made it real, resulting in buildings with a complexity of form and space as well as visual impact.*

Introduction

Over the last thirty some years, Baroque architecture has received new attention from architects, theorists, and students interested in creating what has been called ‘blob’ design: characterised by smooth, complex curvilinear surfaces and volumes, often biomorphic in quality. These qualities are made possible through the use of digital technologies that allow an almost infinite array of formal manipulations, the ‘final’ design essentially only a momentary pause within an endless generative process. Due to their embrace of the Deleuzian Baroque fold, adherents to blob design early on described it as ‘neo-baroque’ and went on to interrogate the Baroque, making it the model and justification for various theoretical and formal ideas that include not only the fold, but also pulsation or rhythm, and glow.[1] But it is the similarities to the complex curvilinear geometries created by 17th century Italian architects that have driven much of contemporary interest in the Baroque. Some have used parametric modeling to analyse Borromini’s buildings as a way of understanding the potential of this tool today, but also go further to suggest that Baroque architects unconsciously employed a form of parametrics. In contrast, others call Baroque architecture ‘messy’ and the result of ‘mistakes’, implying that it was the outcome of erroneous or arbitrary decisions and presenting this as an alternative approach when implementing digital tools to generate unprecedented forms in architecture.[2]

These interpretations of the Baroque, relating to form and process as well as representation, production, and affect, while tantalizing, are not necessarily the outcome of scholarly, historical research. As Delbeke and Leach wrote in 2015, ‘the baroque is a good story for which facts can get in the way’.[3] Furthermore, as stories they are less about ‘quotation’ and more about ‘citation’: the creation of new insights for the production of architecture based on a Baroque subject mediated through the interpretations of philosophers and theorists.

This paper offers an alternative approach based on ‘quotation’ in the sense of breaking the artificial continuity from the Baroque to the present. It does so by considering one issue: the form generation practices of the 16th and 17th centuries. It offers a new historical understanding, based on primary sources and hands-on experimentation with his tools and methods, of how the Baroque designer, using the traditional compass and rule and established geometrical procedures, achieved certain intentions in built form. Although beyond the scope of this paper, these Baroque practices could potentially be used by architects today as a model for how they might think differently about their own tools and methods, as well as their approach to the problem of form.

To understand the form generation practices of the Baroque, this paper begins with the Renaissance treatise of the Sebastiano Serlio, who recorded three geometrical procedures or constructive geometries that had been and would be used for centuries. In order to understand their significance to the early modern architect, these procedures are considered in terms of their earlier history and associated symbolic meanings. In order to understand how they were drawn using compass and rule as well as their inherent formal properties, the sequential steps of each procedure is presented. Next, as the basis for bringing to light how architects used geometrical procedures and the issues they considered while doing so, proposals, established first through drawings using compass and rule, are presented for the steps underlying buildings by Bramante, Palladio, Bernini, and Borromini. Finally, based on a better awareness of these procedures - their implementation, limitations, and potentials - Guarino Guarini’s San Lorenzo is examined. While the steps leading to its complex curves might seem to foresee the indeterminacy of digital design, Guarini’s design and treatise suggests a different approach, one based on a vision created out of a deep knowledge of mathematics, optics, but also buildings of different periods and cultures.

Serlio’s Book I on Geometry

Serlio’s Book I on geometry, first published in 1545, was an essential text for architects for at least two centuries through the many later editions but also treatises that copied his demonstrations. Even in the early 1700s as a student in Rome, Filippo Juvarra redrew many of them.[4] From his own book, we know that Serlio borrowed material from Euclid's *Elementa* (c. 300 BCE), Dürer's *Underweysung der Messung* (1525), and the drawings of his teacher Baldassare Peruzzi (1481-1536). At the same time, because there is evidence from earlier periods of the use of his precepts, it can be concluded that Serlio recorded long-standing and common artisan practices. His Book I presents basic geometrical elements, selected from Euclid, but also methods for generating more complex forms in architecture, including the *biangolo*, quadrature, and the oval (Figure 1).

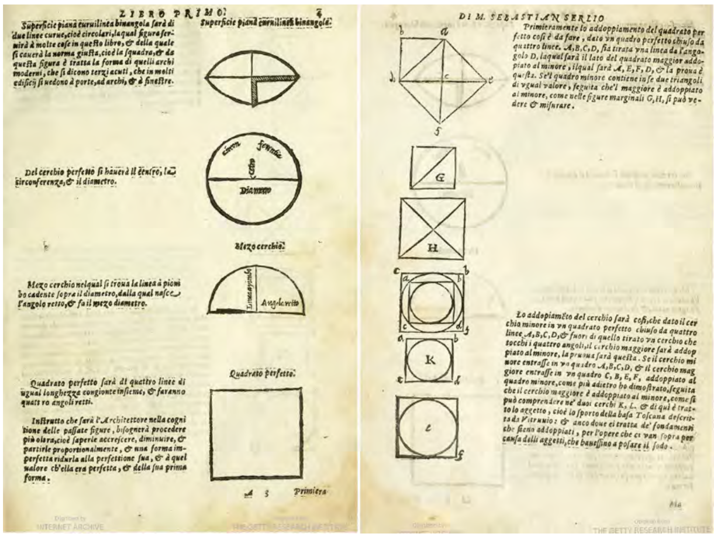

Figure 1. The *biangolo* (upper left) and quadrature (right) as presented by Serlio in Book I. From his *Tutte l'opera d'architettura,* 1619, folios 4-4v. (Courtesy of HathiTrust, https://catalog.hathitrust.org/Record/100236964)

Serlio's 'superficie piana curvilinea biangola' or curvilinear bi-angular plane, taken from Euclid's Book 1, proposition 10, is the most direct method for establishing two lines crossing at right angles, or the 'set square'. By striking of two arcs so that the focus of one is located on the arc of the other, a lens-shaped form results. Called *bisangolo* by Leonardo (c. 1510) and *biangolo* by Juvarra (c. 1709), the construction was also included in the treatises of Francesco di Giorgio (c. 1475), Cesariano (1521), and Dürer (1525). The *biangolo* is useful, Serlio writes, 'for many things' in his book, the simplest being to draw segmental arches for openings. But it is also particularly important as the first steps for drawing the equilateral triangle, either one or two placed base to base, and can be used to construct the oval, as we will see.[5] As such it had broader implications for architectural design. Michael Hill has done a detailed study of the use of the *biangolo* as the generative key in Borromini's design of San Carlo alle Quattro Fontane as well as its symbolic significance in relationship to the Trinity.[6]

Quadrature

Quadrature is presented by Serlio as 'lo addoppiamento del quadro'. (Figure 1) This procedure creates a rotated square with twice the area of the initial square. Serlio goes on to the 'doubling of the circle', based on the principle that a circle inscribed in the larger, rotated square will have double the area of a circle inscribed in the smaller square. Described by Vitruvius (late 1st century BCE), who cites Plato's *Meno*, and drawn by Villard d'Honnecourt (c. 1260-80), the rotating of the square or quadrature was used in architecture from antiquity through the Middle Ages. At the end of the 14th century the term used for this construction was *ad quadratum*. This term also refers to the use of a square grid, as revealed in the Expertises of Milan (1391-1400) that debated the geometry for the cathedral's section.[7]

At the end of the 15th century, the medieval tradition of quadrature was recorded by architects. In his 1486 publication Roriczer referred to the 'old-timers' who practiced this art for the design of gothic pinnacles, but also, as shown by Bork and others, it was commonly used in plan design.[8] Around the same time in his manuscript treatise, Francesco di Giorgio included diagrams of the square grid but also the rotated square for designing churches in the classical style. (Figure 2) Quadrature is also included in the 1511 and 1521 illustrated editions of Vitruvius and was prominently displayed on the frontispiece Serlio's Book I when it was first published in 1545.[9]

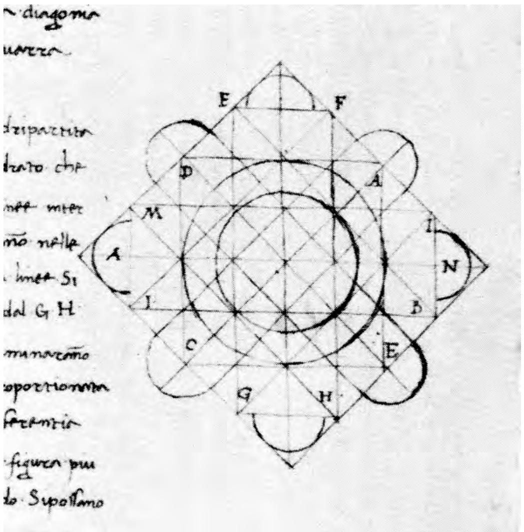

Figure 2. Quadrature as drawn by Francesco di Giorgio c. 1489-92 in his manuscript treatise on architecture, Codex Magliabechiano II.I.141, folio 42. (From vol. 2 of the facsimile edition, Francesco di Giorgio Martini, *Trattati di architettura ingegneria e arte militare*, 2 vols (Milano: Il Polifilo, 1967))

The inscribing of the circle in the square creates infinitely smaller relationships, while the circumscribing of the square around the circle creates infinitely larger ones. Given Vitruvius' description of the man in the circle and the square, drawn first by Francesco (c. 1475), to demonstrate how ancient architects followed nature in creating proportionality among the parts and of parts to the whole, Renaissance architects could well have believed that they were not adapting a method of the 'moderns' to design in the classical style, but rather reviving one used long ago by the ancients. Furthermore, as Wittkower discussed, for Renaissance neo-platonists the Vitruvian man had mystical significance. 'The Christian belief that Man as the image of God embodied the harmonies of the Universe', now combined with Greek mathematics, made the man in the circle and the square 'a symbol of the mathematical sympathy between microcosm and macrocosm'.[10]

While the symbolism of the Vitruvian man may have inspired architects to employ quadrature in design, its practicality made it essential. The primary instruments available were the compass and rule, used by the ancient Romans, and the pencil, an early 16th century invention.[11] Using the tools, the architect drew geometrical procedures, including the *biangolo*, quadrature, and the oval, following step-by-step procedures as the basis for generating a design.

My animation for quadrature shows how beginning with a horizontal line, a circle is drawn to establish points by which to strike arcs for drawing a vertical line (this is the *biangolo* construction) as well as the diagonals. Now the first square is inscribed in the circle and the rotation can begin. Rotating inward requires only drawing lines through stages of intersecting points. Rotating outwards requires using the compass to transfer a circle's radius to become half of the side of a square. Alternatively, half of a square's diagonal is transferred.

These same techniques can achieve quadrature without any rotation: by using a circle to inscribe or circumscribe a square, a series of nested squares is created. But quadrature can also involve the 45 degree rotation of a given square, at the original size. With twin squares rotating inward and outward, the quadrature series achieves a finer grain.

Quadrature results in proportional relationships that are endlessly extendable, an important outcome in the absence of standardised systems of measure. In terms of width, any three sequential squares are related proportionally as 1:√2:2. In terms of area, they relate as 1:2:4. Two steps inside any square is a smaller square that can be quartered and extended to create a grid of 4 by 4, or 16 squares, as seen in Francesco's diagram. (Figure 2) Given the imbedded grid, rectangles based proportionally on 1, 2, 3, and 4 are easily determined. Root rectangles, with incommensurable proportions, can also be created. All of these ratios are contained, according to Alberti (c. 1450), in the room shapes used by the ancients, which Serlio illustrates near the end of Book I.[12] Finally, any set of rotated twin squares inscribe, but also are circumscribed by, an octagon.

By its nature, quadrature lends itself to the centralised plans. Hence, in medieval architecture it was used to design pinnacles, but also towers and some church crossings. During the Renaissance, architects including Francesco di Giorgio, who presented the procedure in his treatise as a tool for design, used quadrature to lay out centrally planned churches for two important reasons. First, as shown by Betts in his study of Bramante's plans for the new San Pietro, quadrature was used to ensure structural stability.[13] Secondly, analysis of Renaissance examples demonstrates that quadrature was used to ensure beauty, through the creation of consistent proportional relationships among all the parts of the design. Particularly important was using the procedure to relate the most visually prominent elements imagined by the architect, one of which would serve as the generative key for the rest of the design. In the plan of the Tempietto (1502-14?) quadrature can be overlaid to coincide with the lowest step and centerline of the encircling colonnade, but there is no alignment with the cella's outer wall or interior space. A more logical alternative is for the quadrature to coincide with the elements that Bramante wanted viewers to see and sense proportionally: the outward faces of the columns, which generates the cella exterior and the outer drum of the dome, and then the face of the interior pilasters.[14] (Figure 3) That the outer face of the circular colonnade was the generative key is confirmed by the built design, which is perceived from the ideal viewing position at the courtyard entry as an object in space. Furthermore, rotating outward locates the round courtyard recorded by Serlio, especially the inside faces of its colonnade, which would be seen in relationship to the Tempietto.

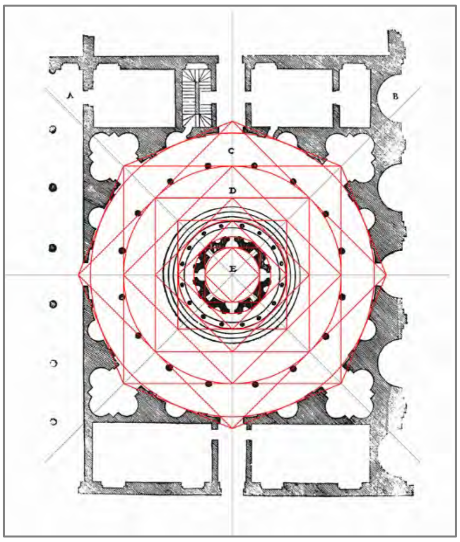

Figure 3. The generation of the plan of the Tempietto and its unbuilt courtyard by means of quadrature. (Author; base plan from Serlio, *Tutte*, 1619, folio 67)

At the Villa Rotonda (1566-70) an important element of the plan is the central domed salon. However, when it is used as the generative key, the outward rotations of the quadrature do not coincide with the square exterior or the projecting porticoes. Alternatively, by rotating inwards from the villa's exterior cubic mass - an essential visual element of the design - the quadrature coincides with the exterior diameter of the dome. More important than the salon's interior, which acts more as a circulation space, the crowning dome was to be seen in proportional relationship to the main cube. Rotating outward, the quadrature determines the location of depth of the porticoes, but not that of their stairs, which hold less visual importance. Architects used quadrature for placing important elements so that the proportional relationships between them, generated by the procedure, would be perceivable, rather than hidden within the mass of columns or walls.

The Oval

In his Book I on geometry, Serlio includes flattened or stretched circles. Although he refers to them all as 'forme ovali', what he presents are two completely different constructions, the ellipse and the oval. The ellipse is a conic section, with a continuous outline constructed by means of specific methods. The oval is a shape formed by two pairs of different arcs: one pair forming the ends and based on two smaller circles; the other forming the longer sides and based on two larger circles.

Although he mentions the gardener's method in Book I and constructs the foreshortened circle in his Book II on perspective, Serlio, who never uses the word 'ellipse', only demonstrates how to draw it using the extended arch method. Whereas in this method only 'a careful, practiced hand', not a compass, can trace the curve between plotted points, he points out that these shapes are the same as 'alcune forme ovali fatto col compasso', that is, the oval construction. Although 'oval forms can be drawn in many ways', he highlights the 'rules' for only four of them.[15] (Figure 4)

Figure 4. Serlio's four ovals. From the 1619 edition, Serlio, *Tutte*, folios 13v-14. (Courtesy of HathiTrust, https://catalog.hathitrust.org/Record/100236964)

Serlio also differentiates between the ellipse and oval in terms of usage. The ellipse should be used in the design of bridges, arches, and vaults with profiles 'flatter than a semicircle'. As for the oval, Serlio infers it should be used in plans. Five years earlier in his Book III on antiquities, he included plans of amphitheaters and a courtyard, describing the latter, as 'in forma ovale molto lunga'.[16] Analysis shows that all are based on oval not elliptical construction. Published two years after Book I, his Book V on churches includes an example based on his fourth oval. In fact, from the Renaissance on, it is oval spaces and domes, not elliptical, which appear in designs, a conclusion shared by most historians.[17] Serlio explains why: 'following the circle in perfection, oval shapes are the next closest'.[18] Constructed using the arcs of four circles, unlike the ellipse, the oval shared the beauty of the circle, and as we shall see, its ease of construction using compass and rule.

Serlio's first 'rule' for ovals is indeed a rule: a diagram showing the common properties of oval construction. (Figure 4, upper left) Two equal, mirror-image, overlapping V's establish four foci. Two are located where the legs intersect and the others at the vertices of the angles. These four foci are the center points for drawing two pairs of arcs that span between the ends of the V's legs, together forming the smooth, curved outline of the oval. The shape of the oval is determined by the location of the center points of the two circles, the radius of those circles, and the location of the two focal points for the side arcs. By maintaining the four anchor points, but changing the radius that generates the arcs, the result will be in, in one direction, a smaller, 'elongated' oval, but in the other a larger more 'circular' one. As the diagram shows, the ovals will always remain concentric to one another, although the smallest one possible will be lens-shaped and the largest will approach but never become a circle. The properties demonstrated in this first oval are universal to all of them.

The four foci resulting from the overlapping V's form a rhombus that establishes the 'geometrical frame' of the oval. Serlio goes on to demonstrate that this frame can be given a specific form and size, creating 'fixed' ovals. The second and third ovals have foci arranged as a rotated square, but one larger than the other. (Figure 4, middle and bottom left) The foci of the fourth oval are arranged as two equilateral triangles placed base to base - the *biangolo* discussed earlier. (Figure 4, top right) Many ovals can be created from a given geometrical frame, but Serlio's are based on those circles that initially establish the end arcs and the frame. The second oval is 'very similar to a natural egg', with tapered ends, and was called *ovato longo* by later writers. The fourth has ends that are flatter, but sides that are more curved. For its roundness, it became known as *ovato tondo*. It also has 'dolcezza', Serlio writes, being easily drawn in four steps, as opposed to the eight required for the others.[19]

Serlio's knowledge of the general properties of the oval came directly from Dürer and Peruzzi. In a discussion of the profile of ancient vases, he gives examples based on ellipses, but also includes a simplified version of Dürer's construction of an 'egg-shaped line', an irregular oval or ovoid.[20] Among Peruzzi's drawings are free-hand sketches of Serlio's so-called third and fourth oval, but also images, constructed using compass and rule, of the amphitheater at Verona and various oval designs based on the fourth oval.[21] Peruzzi's own teacher, Francesco di Giorgio, in his c. 1475 treatise drew the Colosseum, describing it as 'a fforma di huovolo cioè più longo che largho', as well as rooms, courtyards, and vault types 'a huovolo'.[22] Knowledge of oval construction probably was received from the Middle Ages - evidence of it is found in the profiles of cross vaults and in the plan of domes - but originated in the design of ancient Roman amphitheaters.[23]

The usefulness of fixed ovals, first recorded by Peruzzi and then published by Serlio, led others to copy his diagrams but also to develop other types. Around 1550 Vignola drew the so-called commensurable oval with a geometrical frame of four Pythagorean triangles. In his 1629 treatise, Viola Zanini proposed an oval generated from two squares overlapping at their center points.[24] Like Serlio's, these new fixed ovals are easy to draw; Vignola's because its frame is created by measured units rather than geometry. Furthermore, these ovals are associated with certain proportions. Although any geometrical figure can be inscribed in an oval, Serlio's third oval is presented with a double square inscribed and Viola Zanini's a square plus one half. Since neither one is constructed using these shapes, their inclusion indicates a desire to demonstrate how these ovals harmonise with Alberti's classical room proportions, a feature we saw embedded in quadrature.

Despite their name, fixed ovals are <u>not</u> fixed: their shape can easily be changed by manipulating the geometrical frame. One reason architects did so was to create greater unity between the central oval and its surrounding envelope of spaces. In Volterra's scheme for San Giacomo Incurabili (1590), which includes entries on the short axis, the V's of the geometrical frame match the sight lines into the center of the side niches, creating an ideal coordination between constructive geometry and spatial/visual experience.[25] This not the case in Bernini's first design for Sant' Andrea al Quirinale (1658-70), as Smyth-Pinney has demonstrated.[26] In it the center is laid out based on Serlio's second oval and is surrounded by a complex pattern of chapels, but there is no coordination between the geometrical frame and the centerlines of openings or pilasters. The same is true in Bernini's second scheme, where the initial circles of the oval are enlarged, changing its geometrical frame while the original length and width is maintained. Finally, by enlarging the circles even further, the new oval's geometrical frame aligns with the pilasters, and its foci organise the sight lines into the chapels. By manipulating the oval's constructive geometry, Bernini created greater unity between the visitor's unconscious experience of its geometrical frame and his visual experience of the surrounding chapels and pilasters. The morphological series in Figure 5 demonstrates how easily the shape of Serlio's second oval be manipulated using compass and rule.

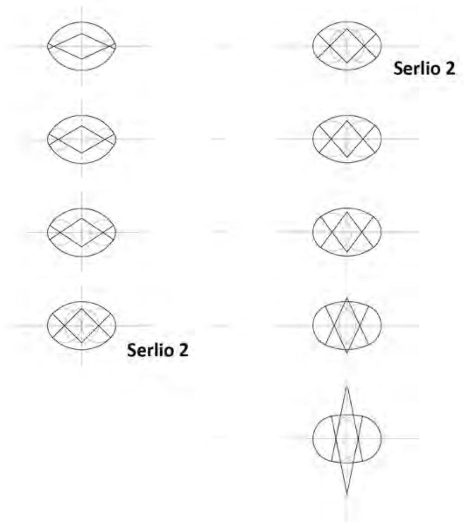

Figure 5. Serlio's second oval reshaped: the radius of the two initial circles changes, but the length and width are fixed. (Author)

San Carlo alle Quattro Fontane (from 1634) is another example of how an initial oval can be reshaped. The discrepancy between the oval construction used to generate the plan and the dome's appearance today has been long recognised. Hill has demonstrated that the plan originated with the *biangolo*, two equilateral triangles, and an inscribed oval.[27] Taking the *biangolo* oval as a starting point, an exploration of its morphology reveals how by shifting the geometrical frame, even with the length and width fixed, the shape can be changed considerably. In one direction the oval ends become more and more pointed, but in the other they become rounder and the sides flatter, as can be observed in the dome as built.

Guarini's San Lorenzo

Upon receiving his first commissions in 1660, Guarini took up the same simple drawing tools and geometrical procedures used by architects, both past and present. But as a mathematician and astronomer, he was already adept in using them. Guarini published a commentary on Euclid in 1671, which includes sections related to the *biangolo*, quadrature, the oval, and the ellipse, and went on to incorporate these and other geometrical principles necessary to architecture in his treatise *Architettura civile*, begun during the early 1670s and published posthumously (1737).[28]

Given the tools and procedures available to him, how did Guarini create the complex curvilinear forms found in buildings such as San Lorenzo (1668-87)? Based on what has already been discussed, the first step is to identify a prominent visual element as the generative key. Outside there is no distinct formal expression; inside the swelling walls and misaligned columns are geometrically elusive. But the clarity and visibility of the circular cornice of the dome suggests it plays an important role in a quadrature series. That circle at A (step 1) is expanded outward three steps to establish two rotated squares at Stage D. (Figure 6)

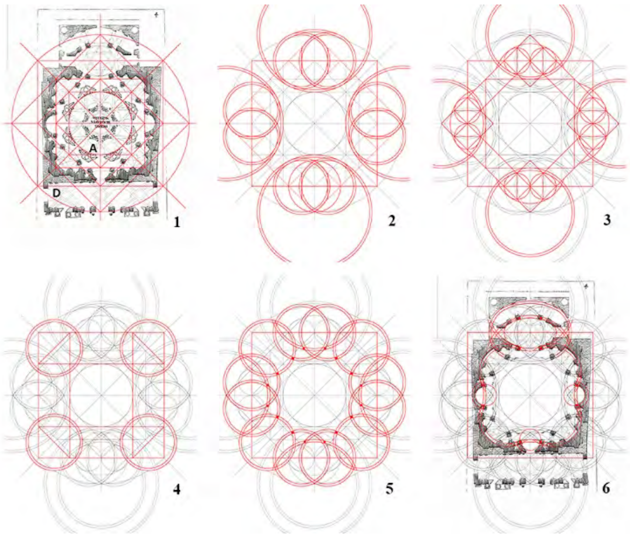

Figure 6. The generation of the plan of San Lorenzo (Author; base plan from Guarinio Guarini, *Architettura civile*, 1737, plate 4)

Next, Serlio's second oval is constructed at the four tips of the rotated square to generate the complex shape of the perimeter spaces and enclosing walls. Within the four ovals the same foci are used to draw smaller ones. In step 4, the circle used to define each end of the large ovals is now transferred to the corners and centered on the four corner squares of the 16 square grid inherent within quadrature. Next, the eight column pairs are placed. On the cross axes they are located along the curve of the small oval. At the corners they are placed where the corner circles overlap with the identical circles of the large oval. Finally, the outline of the wall surface is defined. At the ends of the cross axis and at the entry, small circles are drawn centered on the curve extending between the two columns of the small oval, creating two niches and a vestibule respectively. In each corner, the curved outline of the columns is reflected opposite in the curve of the wall. Viewed in relationship to the curved column pairs on the cross axes, the plan undulates.

Whereas the design of San Lorenzo could be interpreted as the result of a series of choices which could just as well have gone in other directions, Guarini's own theoretical statements suggest otherwise. In his writings, he sets forth the platonic concepts of *idea* and *disegno*. The 'idea' is created by the architect in the mind; the 'drawing' imitates or records it. But essentially they are one and the same: the architect's mind and hand working together so closely that the process of forming, developing, and perfecting the idea is the same as drawing it, and vice versa.[29]

For Guarini, the *idea/disegno* - the architect's vision - must be shaped by knowledge and experience. This meant upholding the classical tradition, but also breaking from it. He states that architecture 'is based on Mathematics': the geometries and proportions associated with the classical style, not just the simple volumes but their complex intersections. But he continues: 'it nonetheless an art that delights, which does not want to displease the senses for the sake of reason: … when those [mathematical] demonstrations threaten to offend the sight, they are changed, abandoned, and finally even contradicted'. Due to the inability of the eye to perceive precisely, resulting in optical illusions, the geometry must be changed: 'To preserve the proper proportions in appearance, architecture must depart from the rules and from true proportions'.[30]

Guarini, however, expresses interest in the potential of optical illusions, including those achieved through non-classical forms. He admires Gothic architecture, which he knew first-hand from his travels through Italy, France, and Spain. The 'ingenious' builders, by creating strong buildings that 'seemed weak and whose standing up seemed miraculous', were able to 'astound the intellect and terrify the spectators'. In these buildings is found great height in relationship to width. Cupolas and towers are raised high on slender supports or placed on arches or on top of vaults. Arches 'bend' from their bases and 'hang in the air'. There are 'perforated towers', 'elevated windows', and 'vaults without sides', suggesting he saw the Islamic openwork domes at Cordoba.[31] Looking at San Lorenzo, it is clear that these same qualities and elements contributed to Guarini's vision from the beginning.

The design of San Lorenzo, with its curvilinear and intersecting geometries, three-dimensional arches, ribbed and layered domes, and extremes of light that create the effect of infinity, weightlessness, and dematerialisation, was the result of a deep knowledge of mathematics and optics, as well as direct experience of the architecture of all periods and cultures then known. The architect formed an idea in his mind and, drawing with compass and rule, worked with geometrical procedures to perfect his vision through multiple reciprocal iterations. Proceeding with knowledge and intent rather than indifference, controlling his tools and methods rather than giving them free rein, the Baroque architect created his vision and made it real. The result are buildings with a complexity of form and space as well as visual impact that continue to be admired but also emulated by architects today. For those seeking to create such geometries and effects using new technologies, a historical understanding of Baroque form generation practices may provide valuable insights.

Endnotes

[1] Mario Carpo, 'Digital Style', *Log*, 23 (2011), 41-52. Michael J. Ostwald, 'Ethics and Geometry', in Sylvie Duvernoy and Orietta Pedemonte (eds.), *Nexus VI, Architecture and Mathematics* (Torino: Kim Williams Books, 2006), 77-92.
[2] Andrew Saunders, 'Baroque Parameters', *Architectural Design*, 79, 1 (2009), 132-5. Cameron Wu, 'Of Circles and Lines', *Log*, 31 (2014), 107-14.
[3] Maarten Delbeke and Andrew Leach, 'The Future of the Baroque, c. 1980', in Andrew Leach, et al. (eds.), *The Baroque in Architectural Culture, 1880-1980* (Farnham, Burlington: Ashgate, 2015), 234.
[4] Salvatore Boscarino, *Juvarra architetto* (Roma: Officina, 1973), 145-6.
[5] Hart, *Serlio*, 8. Augusto Marinoni, 'The Writer', in Ladislao Reti (ed.), *The Unknown Leonardo* (New York: McGraw-Hill, 1974), 74. Boscarino, *Juvarra*, pl. 79. Francesco di Giorgio, Codex Saluzziano 148, Torino, f. 30v; this manuscript is reproduced in volume 1 of Francesco di Giorgio Martini, *Trattati di architettura ingegneria e arte militare*, 2 vols (Milano: Il Polifilo, 1967). Cesare Cesariano (ed.), *Di Lucio Vitruvio Pollione De architectura libri dece* (Como: Gotardus da Ponte, 1521), Book III, XLIX-XLIXv, at the feet of the Vitruvian man. Euclid, *Elements*, Book I, prop. 10 (to bisect a line), prop. 1 (to construct equilateral triangle). Albrecht Dürer, *The Painter's Manual*(New York: Abaris, 1977), Book I, 76-7 and fig. 21; Book II, 148-9, fig. 18. Erwin Panofsky, *The Life and Art of Albrecht Dürer* (Princeton: Princeton University Press, 1955), 245. In Book II, fig. 18. Dürer calls this form by its artisan name "Fischblase" ("fish's bladder"), translated in Latin editions (1532, 1606) as *vesica piscis*.
[6] Michael Hill, 'Practical and Symbolic Geometry in Borromini's San Carlo alle Quattro Fontane', *Journal of the Society of Architectural Historians*, 72, 4 (2013), 555-83.
[7] Hart, *Serlio*, 9. Vitruvius Pollio, *On Architecture*, 2 vols (Cambridge, London: Harvard University Press, W. Heinemann, 1931), II, 199, 201 (Book IX, preface, para. 4-5). Villard de Honnecourt, Magister 2, f. 20r. See Carl F. Barnes, *The Portfolio of Villard de Honnecourt* (Burlington: Ashgate, 2007), 134-36, diagrams labeled k and o; Paul Frankl, *The Gothic* (Princeton: Princeton University Press, 1960), 50-54, 68-71.
[8] Lon R. Shelby, ed., *Gothic design techniques: the fifteenth-century design booklets of Mathes Roriczer and Hanns Schmuttermayer* (Carbondale: Southern Illinois University Press, 1977). Robert Bork, 'Dynamic Unfolding and the Conventions of Procedure: Geometric Proportioning Strategies in Gothic Architectural Design', *Architectural Histories* 2, 1 (2014), DOI: http://dx.doi.org/10.5334/ah.bq.
[9] Francesco di Giorgio, Codex Magliabechiano II.I.141, f. 42-42v; this manuscript is reproduced in volume 2 of Martini, *Trattati*. Also drawn by Fra Giovanni Giocondo (ed.), *M. Vitruvius per Iocundum solito castigatior factus* (Venetiis: Ioannis de Tridino alias Tacuino, 1511), 84 and by Cesariano, *Vitruvio*, CXXXXIIII.
[10] Vitruvius, *Architecture*, I, 161 (Book III, Ch. 1, para. 3-4). Francesco di Giorgio, Codex Saluzziano 148, f. 6v; Giocondo, *Vitruvius*, 22; and Cesariano, *Vitruvio*, L. Rudolf Wittkower, *Architectural Principles in the Age of Humanism* (New York: W. W. Norton, 1971), 13-19.
[11] See Maya Hambly, *Drawing instruments, 1580-1980* (London: Sotheby's, 1988).
[12] Leon Battista Alberti, *On the Art of Building in Ten Books* (Cambridge: MIT Press, 1988), 306-7 (Book IX, Ch. 6). Hart, *Serlio*, 30.
[13] Richard J. Betts, 'Structural innovation and structural design in Renaissance architecture', *Journal of the Society of Architectural Historians*, 52, 1 (1993), 5-25.
[14] My ideas are derived from the study of proportions in Mark Wilson Jones, 'The Tempietto and the Roots of Coincidence', *Architectural History*, 33 (1990), 1-16.
[15] Sebastiano Serlio, *Tutte l'opere d'architettura, et prospetiva* (Venetia: Giacomo de' Franceschi, 1619), 11v. Hart, *Serlio*, 22.
[16] Serlio, *Tutte*, 58. Hart, *Serlio*, 114.
[17] For example, Sylvie Duvernoy, 'Baroque Oval Churches: Innovative Geometrical Patterns in Early Modern Sacred Architecture', *Nexus Network Journal* 17, 2 (2015), 425-56; but not George L. Hersey, *Architecture and Geometry in the Age of the Baroque* (Chicago: University of Chicago Press, 2000), 133-44 (Ch. 6, Stretched Circles and Squeezed Spheres).
[18] Hart, *Serlio*, 402-3.
[19] Hart, *Serlio*, 26-7. Timothy K. Kitao, *Circle and Oval in the Square of Saint Peter's* (New York: New York University Press, 1974), 31.
[20] Hart, *Serlio*, 24-5. Dürer, *Painter's Manual*, 77-8, fig. 22.
[21] Uffizi 531 a r. See Heinrich Wurm, *Baldassarre Peruzzi: Architekturzeichnungen* (Tübingen: Wasmuth, 1984).
[22] Francesco di Giorgio, Codex Saluzziano 148, f. 71, 20, 21v.

[23] Santiago Huerta, 'Oval Domes: History, Geometry and Mechanics', *Nexus Network Journal*, 9, 2 (2007), 218-23. Mark Wilson Jones, 'The Setting out of ampitheatres: Ellipse or Oval?', in Tony Wilmott (ed.), *Roman Amphitheatres and Spectacula* (Oxford: Archaeopress, 2009), 5-14.
[24] See Kitao, *Circle*, 34. Vignola's commensurable oval first appears in Parma, Archivio di Stato, Mappe e Disegni, vol. 49, n. 17. Giuseppe Viola Zanini, *Della architettura* (Padova: Francesco Bolzetta, 1629), 26-7.
[25] Volterra's drawing is Nationalmuseum, Stockholm, NMH CC 2071. Duvernoy, 'Baroque Oval Churches', 445.
[26] Julia M. Smyth-Pinney, 'The Geometries of S. Andrea al Quirinale', *Journal of the Society of Architectural Historians,* 48, 1 (1989), 53-65.
[27] Hill, 'Practical and Symbolic Geometry', 558. See Marco Canciani, 'Il disegno della cupola del San Carlino alle Quattro Fontane di Borromini: ovale canonico?', *DisegnareCon*, 8, 15 (2015), 12.1-12.22.
[28] Guarino Guarini, *Euclidis adauctus et methodicus* (Augustae Taurinorum: B. Zapatae, 1671) and *Architettura civile*, ed. Bianca Tavassi La Greca (Milano: Il Polifilo, 1968).
[29] Guarini, *Architettura*, Tract I, ch. 1, 9. See Bianca Tavassi la Greca, 'La posizione del Guarini in rapporta alla cultura filosofica del tempo', in Guarini, *Architettura*, 439-59.
[30] Guarini, *Architettura*, Tract I, ch. 3, introduction, 10-11; obs. 7, 17.
[31] Guarini, *Architettura*, Tract III, ch. 13, obs. 1, 207-10.