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Pythagorean Palladio: Palladian Proportionality Patterns Decoded?

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Keywords

Palladio
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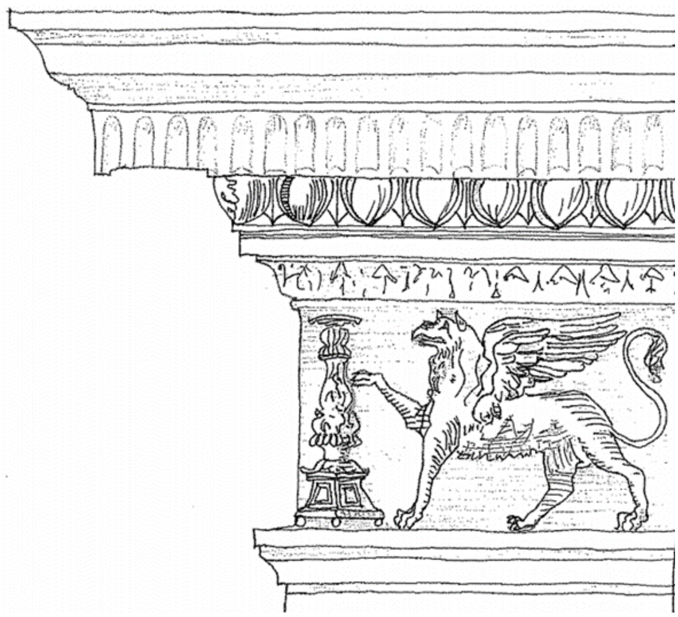
Root two rectangle
Tetraktys
Pythagorean Lambda
Renaissance architectural theory of proportions
Palladian “geometric” method
Palladian “harmonic” proportions

Abstract

There seem to be many understated assumptions in Palladio’s Four Books on Architecture. He tells us what to do, but never fully explains in detail how he gets to the recommended methods. For instance he uses drawings that understate the use of the root two rectangle (Figure V) in the first geometric method suggested to calculate the heights of vaults and uses the numbers of the tetraktys – without mentioning it - to “calculate” and provide an example of “harmonic” proportion. Had he only been a little more specific many misconceptions about his work – as being not entirely clear and somehow mysterious- might have never eventuated. In this brief work we show Palladio’s use in his designed and built work of one of his favourite proportions, the “proportione diagonea” already mentioned by Serlio, none other than the root two rectangle - the root two being, according to Wittkower, in his 1949 seminal work, the only irrational number of relevance in Renaissance architectural proportionality theory. A brief explanation of how the Tetraktys and Pythagorean Lambda were allegedly used by the stonemason turned architect to “calculate” “arithmetic”, “geometric” and “harmonic” means is provided.



Introduction

Renaissance Italy was a melting pot of ideas and theories, circulated and discussed in elite intellectual circles by a lucky few. Palladio, initially a stonemason, was one of them, based not on his family of origin but on his, most probably always obvious, talent. The noblemen Trissino and Barbaro had been his mentors and the result was the marvel of synthesis and sophistication that we all know as the father of Palladianism. Palladio collaborated on the translated version of Vitruvius' ten books by Barbaro and, in doing so, gained useful and fruitful insights into the classical approach to his subject matter, architecture.

An issue with Palladio's Treatise The Four Books on Architecture has always been the little amount of explanation that he gives about the recommended methods to calculate the heights of rooms, in particular vaulted ones. These are best placed at the lower floors of the building, as being vaulted they would be aesthetically pleasing and better suited in case of fires. Whether this has to do with Palladio's education having been less rich than that of others, for instance Bertotti Scamozzi, as suggested by Ortolani (Ortolani, 2015), or simply due to his assumption that people could understand what he meant just by getting the succinct, in his opinion sufficient, amount of information he provided, is not clear at present to the author.

The fact remains that, by not providing the actual heights of his buildings, Palladio has left readers, for nearly the last five centuries, often confused by his methods to calculate them. They all have fallen into the unified category of "harmonic" proportions, although there are more of them other than the "harmonic proportions" method itself, as we will revise briefly in this work.

The paradigmatic Rotunda Villa by Palladio, a suburban (at the time) Villa, although being residential, resembles somehow, in elevation, the Pantheon in Rome. It is a symmetrical, cubic, villa, with a small dome and hexastyle porticoes on all its (inter-Cardinal) four facades. In some authoritative authors' opinion, like for instance Unwin in "Analysing Architecture" (Unwin, 1967), Palladio's work in the Rotunda "reads" easily in plan – like "ad quincunx" squares- but is difficult to read in elevation. Let us try and prove our eminent predecessors wrong and see how – given the right clues to decode also the proportionality patterns of the elevation of the famous Villa – the overall "reading" becomes easy or, at least, easier.

The Rotunda Villa: Built Based on the Palladian First "Geometric" Method?

In the Palladian masterpiece, the Rotunda (in Vicenza, Italy), is the fact that Palladio's drawing for the first geometric method to determine the heights of vaults – by finding, geometrically/with drawings – in the manner of German Renaissance painter Dürer - the golden mean between width and length of a rectangular room (therefore somehow "squaring" that same room and "finding" a gnomon that can morph into a root two rectangle according to Elam's method of transformation, in some way also similar to some of Serlio's drawings, see Figure S) - contains also a root two rectangle only a coincidence of the geometric construction? If that were to be the case, wouldn't the root two rectangle measurements in the Rotunda central part around the circular hall, identified by March (March, 1998), not be somehow proof that the method chosen for and used, by Palladio, in his Rotunda had been the [first] "geometric" one? The side rooms of the Rotunda have been stated by Mitrovic (1990) to have heights determined with the "arithmetic" method – the same opinion that March had about the Villa, in a note about the central circular hall's height, but is that so? The author has doubts about it[i].

i. The fact that the first geometric method gets so often confused for the arithmetic method is an understandable misunderstanding because the architect refers to it as the first method, engendering confusion in the reader - see also page 149 MIT where Palladio, about another building, says "on either side there are two rooms a square and two thirds long **whose height was set according to the *first* method of establishing vault heights**"{emphasis added}- "first" method that, numbers [of Vicentine feet in plan] at hand: 27,12,18, turns out to be {given √**27x12**=√324=**18**} the "first" geometric one?

In Mitrovic's 1990 paper, the unsolved mystery of the Rotunda's proportionality analysis is mentioned when he writes that "we are at the same time so hopelessly unsuccessful in our attempts to explain the Rotunda's proportions on the basis of the theory of harmonic proportions, or in any other way" (my emphasis). According to the Appendix Palladiana in the book "Portrait of a Bridge…" (2019), by the author, the Rotunda Villa - displaying a tri-axial "symmetry"- is spectacularly based in terms of proportionality patterns on root two rectangles used by Palladio assumedly in a "geometric" method fashion (according to the first geometric method he proposes at Chapter XXIII of Book I). The Villa is seen as an attempt of "doubling", or simply building, the cube spatially like Dürer or Barbaro would have done, so built based on the "doron" principle, a la Barbaro, and/or on the Dürer method (ex 'Der Messung…"/"The Painter's manual…", printed in Nuremberg in 1525). See figure D (from figure 44 of book 4 of "The Painter's Manual…", ex March, 1998).

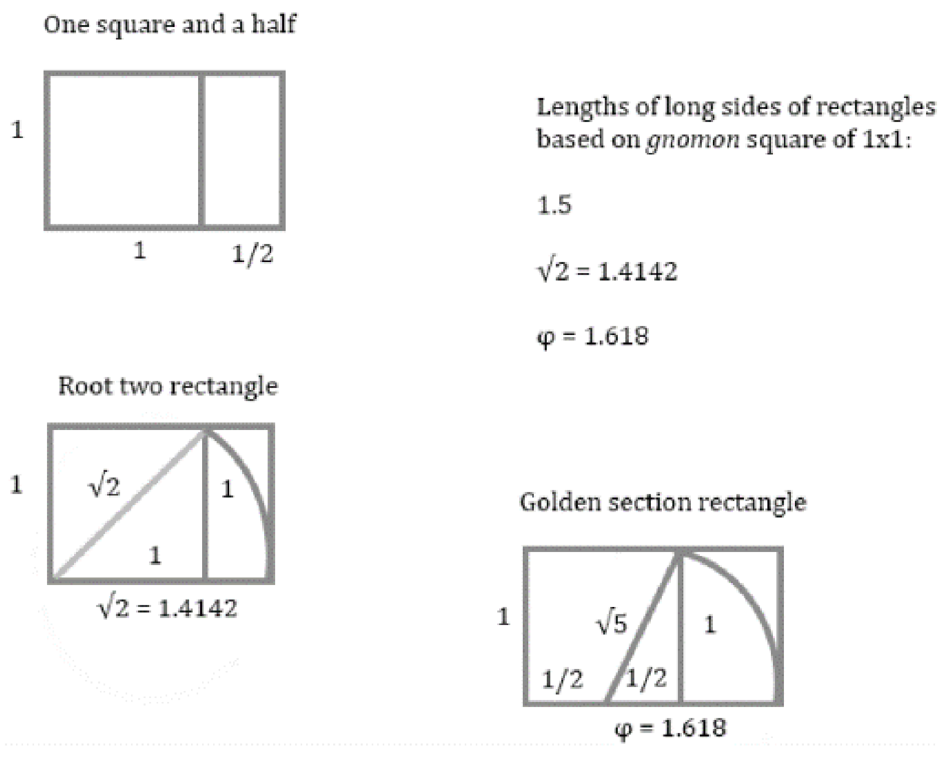


Figure R above: rectangles given by one and a half square, a Root two rectangle and a Golden Section Rectangle

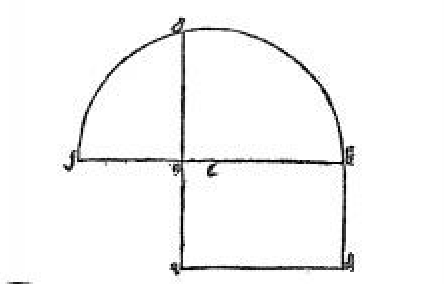


Figure G: First "geometric" method suggested by Palladio to calculate Heights of vaults

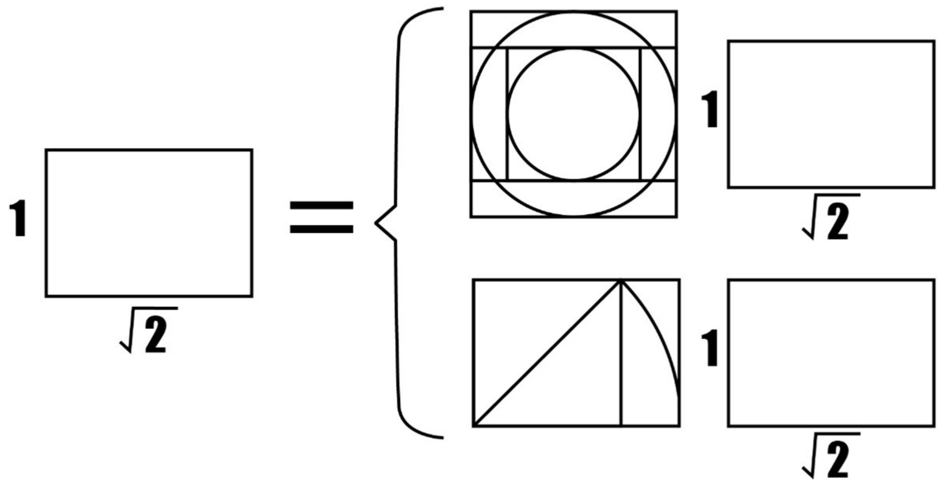


Figure S: geometric construction of Root two rectangle – upper drawing after Elam, 2001 (after Serlio?) and lower drawing after Serlio (in March, 1998), source: "Portrait of a bridge…", 2019.

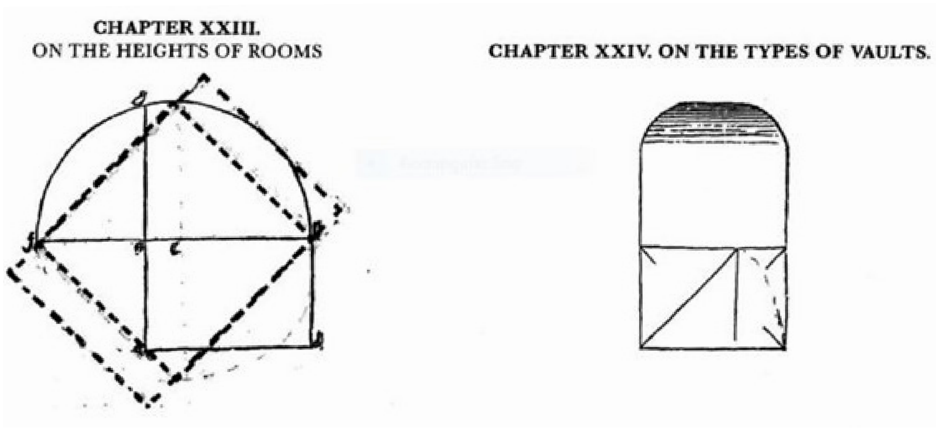


Figure V: figures on vaults from Palladio's Four Books on Architecture; additional dotted lines, by author, reveal the root two rectangles understated in Palladio's drawings (Source: "Portrait of a bridge…", 2019, by author).

Palladio assumedly also uses the root two rectangle as a proportionality tool in other buildings other than the Rotunda Villa – The Villa Almerigo Capra in Vicenza – see figures 1 and 2. In the essay "Villa Poiana a struggle for the ideal", by Daniel Ayad it is stated that in the Villa Poiana: "The outline of the central bay, on the other hand, is equivalent to the ratio of a root-square; the loggia in this instance being the root (sic) added onto the perfect square."(emphasis added by author)

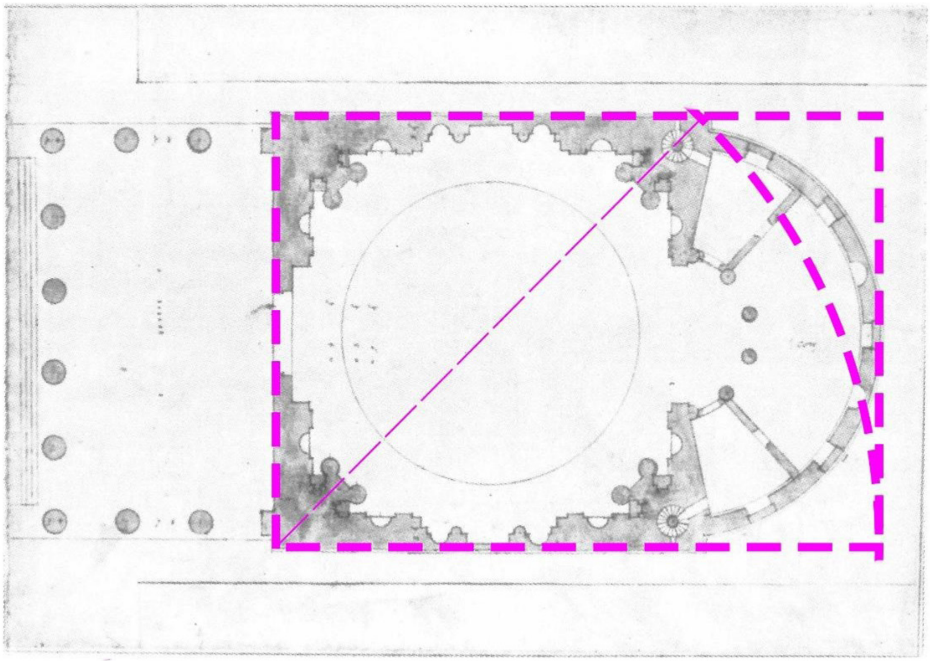
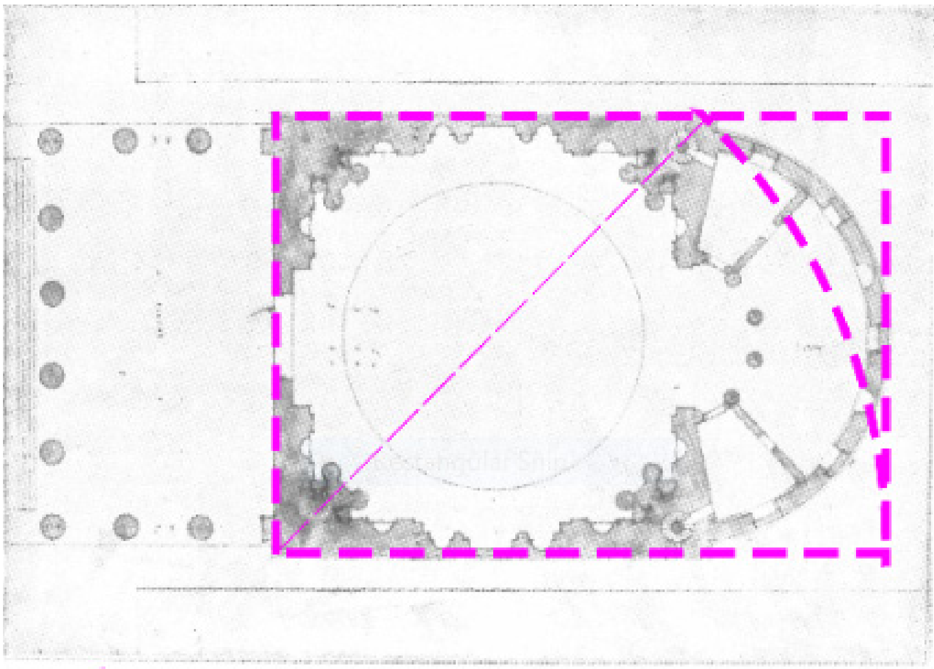


Figure 1: (graphic work courtesy of Ximena Parsons): Plan for the design by Palladio of a Church (*)[ii] in Venice – 1579 – also inscribed in a "proportione diagonea", a root two rectangle

(*) =according to Wittkower in 1949 ("Architectonic Principles in the age of humanism", ex figures 40a and 40b) the San Nicola da Tolentino Church; according to the RIBA website, accessed in March 2020, the Redentore Church, also in Venice. RIBA Collections.

ii. (*) =according to Wittkower in 1949 ("Architectonic Principles in the age of humanism", ex figures 40a and 40b) the San Nicola da Tolentino Church; according to the RIBA website, accessed in March 2020, the Redentore Church, also in Venice. RIBA Collections.



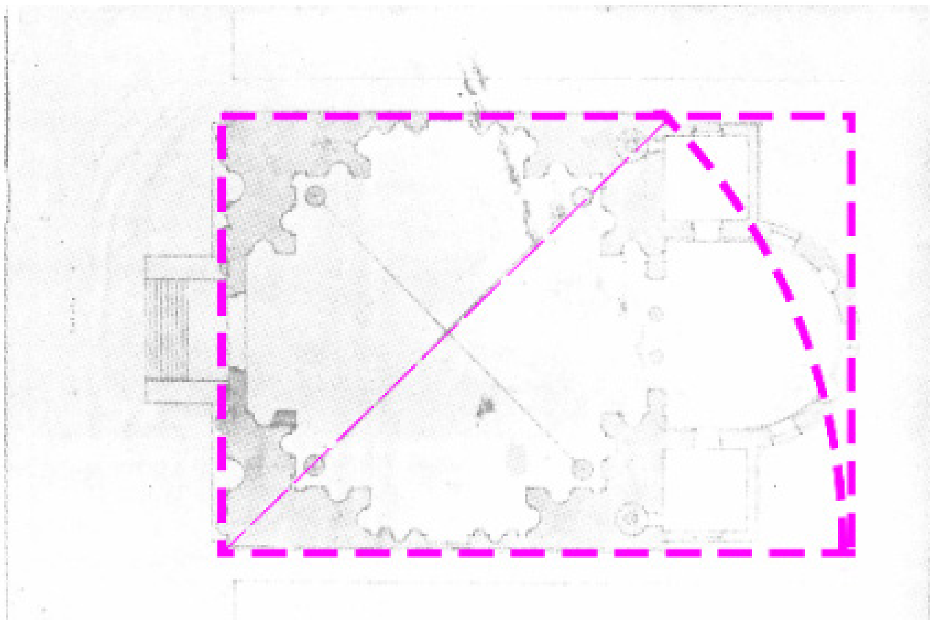


Figure 2: (graphic work courtesy of Ximena Parsons): after Wittkower, 1949 ("Architectonic Principles in the age of humanism", ex figures 40a and 40b) the San Nicola da Tolentino Church plans (upper one same as figure 1) by Palladio, that both fit in a root two rectangle proportioning tool. RIBA Collections.

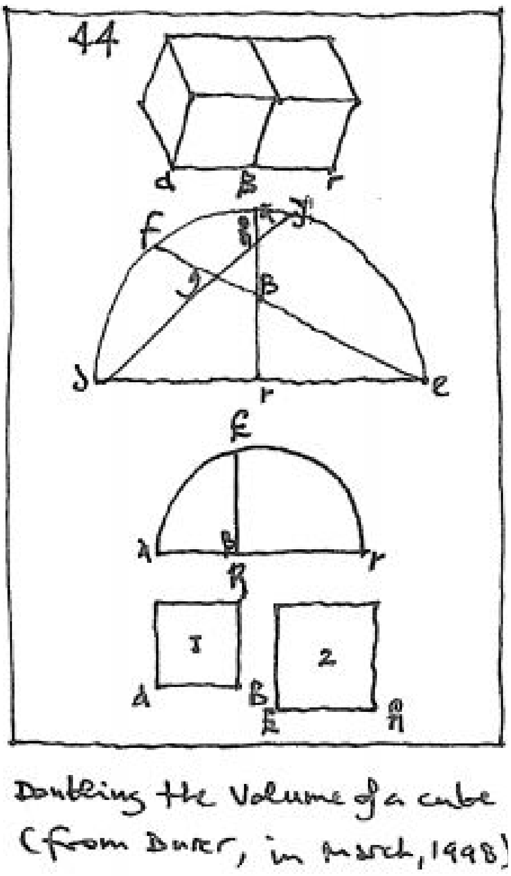


Figure D

With the Doron principle, we imply that the dimensions of the Villa ought to be the same "*in every way*" (March, 1998),the puzzling "*quoque versus*" found in Vitruvius's Ten Books, that Morgan, in 1960, as reported in a note in Schofield's Penguin later translation, translates as "*along all axes*". The same dimensions in that case turn out to be root two rectangles based on the gnomon that is at the centre of the symmetrical building – we repeat symmetrical along the x and y axes and, furthermore, although unbeknown to most of visitors and scholars, most probably also along the z axis (at least based on the drawings and not also on the as-built building itself - until a 3D meticulous relief/survey is executed to duly check it out). We fully agree with Mitrovic (1990) when he states the following: "We may conclude that, although the theory of harmonic proportions accounts for most of the proportions in Book II, it does not account for all of them, nor it is the only theory used by Palladio" (emphasis added by author). With Mitrovic's other statement (also in Mitrovic 1990) that "Whether Palladio ever used any such 'principle of interproportionalization' is still an open question" we do not however agree. While Mitrovic goes on to formulate an original hypothesis about it, in his own words "plausible to a certain degree", we frame an altogether different one, hopefully plausible as well. It originates from Cornford's statement (1998), that truncating the Pythagorean Lambda to the cubes had nothing to do with musical harmony, it had to do with aiming at tridimensional solidity. In Wittkower's words[iii] "In contrast to France and England, most Italian monumental architecture is cubic and conceived in terms of a solid three-dimensional block. Italian architects always strove for an easily perceptible ratio between length, height and depth of a building, and all villas by Palladio have that block-like quality" (emphasis added by author).

iii. in *Principles of Palladio's Architecture*, 1944, page 111.

When we state that the ideology – and related building measurements - behind the Rotunda Villa building could have been one method and/or the other, meaning the first Palladian "geometric" method, after Dürer, somehow used to double the cube in the Rotunda, and/or the "doron" so-called (by author of this work) principle as found in Barbaro's translation of Vitruvius and understood from there, it is because we note that in the second drawing from the top of the figure 44 of book 4 of Dürer's "The Painter's Manual, with compass and ruler" (Nuremberg, 1525), both a golden section rectangle (placed vertically) and a root two rectangle (placed horizontally) fit in it, with one side of the "gnomon" adjacent to the vertical radius line of the semicircle DE, perpendicular to line DE. Therefore, in terms of proportionality analyses, the doubt remains as whether the design principles of the Rotunda were based on merely following, after Barbaro, the "doron" principle – with root two rectangles' measurements "in every way" (March, 1998) – or also on trying to somehow spatially "double" the central round hall's enveloping "small" (internal to the building) "virtual" cube, by using the [first] "geometric" method construction, following Dürer.

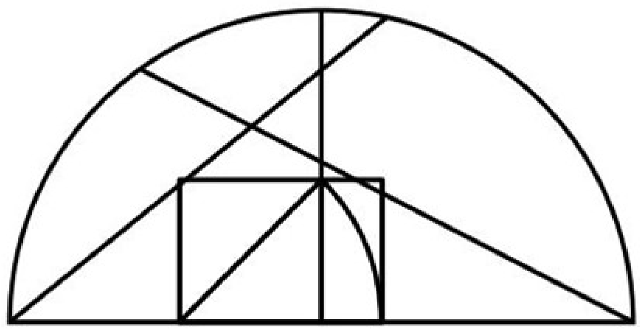


Figure D1 sketch by author ("Portrait of a bridge…", 2019): horizontal root two rectangle in Durer's fig.44 book 4

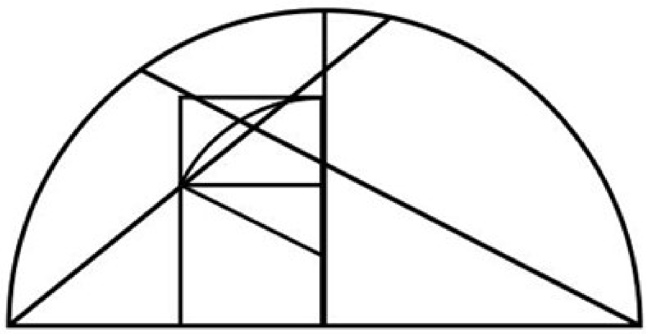


Figure D2 sketch by author ("Portrait of a bridge…", 2019): vertical golden section rectangle in Durer's fig.44 book 4

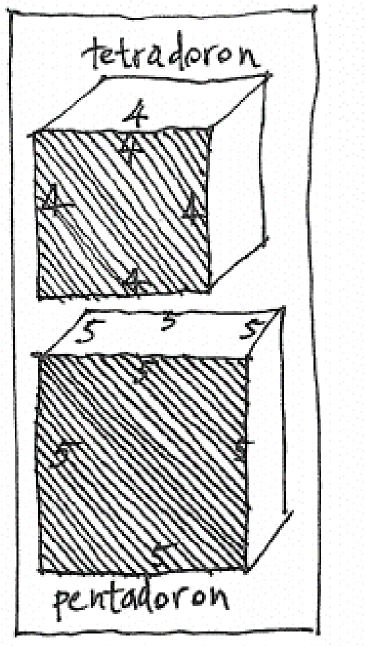


Figure showing tetradoron and pentadoron cubic bricks, an early solution proposed by Barbaro to the Delic problem[iv] of doubling the cube, ex March, 1998

iv. The Delic problem is one of the three "classical" mathematical problems: the one about how to double the volume of a cube. Doubling the side of one side of the cube will not double its volume. It was originally investigated by the ancient Greeks, to double the volume of the altar of the sacred temple of Delos (for the Gods to have mercy on the local population), hence the name.

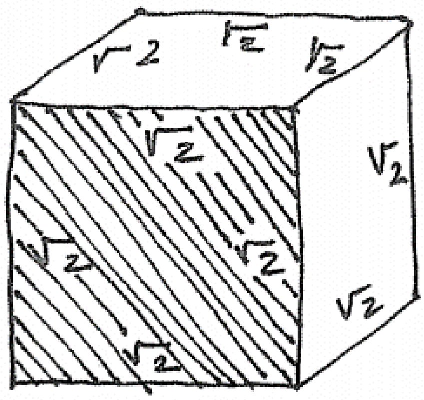


Figure showing a cube based on the root two rectangle "in every way" (March, 1998), "along all axes?" (Morgan 1960), sketch by author ("Portrait of a bridge…", 2019)

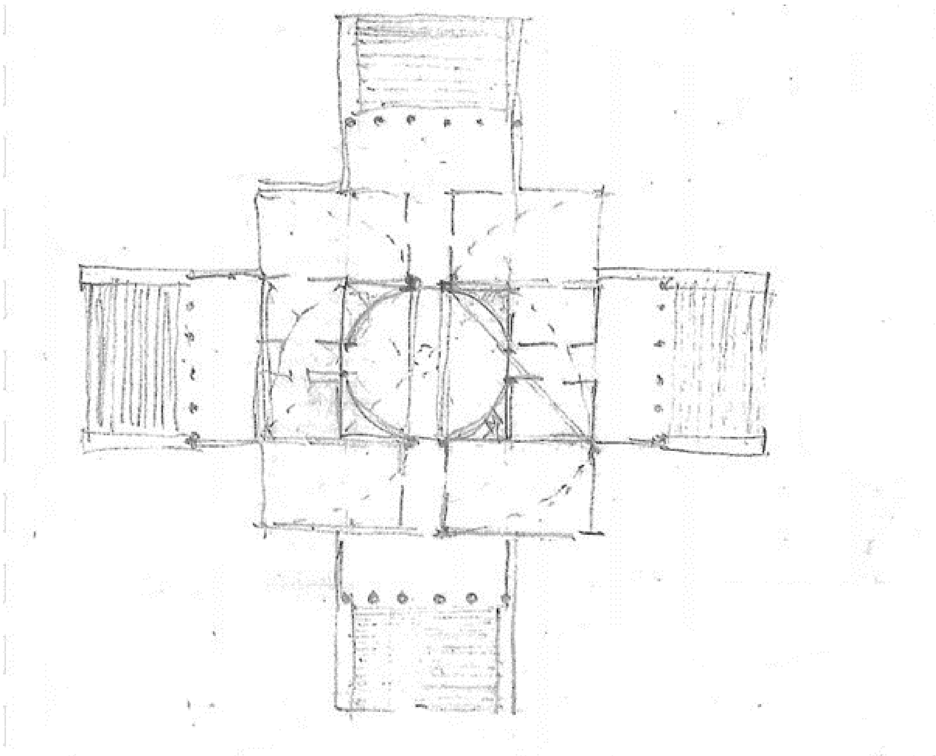


Figure P1: root two rectangles proportioning tool for the plan of the Rotunda, from a sketch by author ("Portrait of a bridge…", 2019)

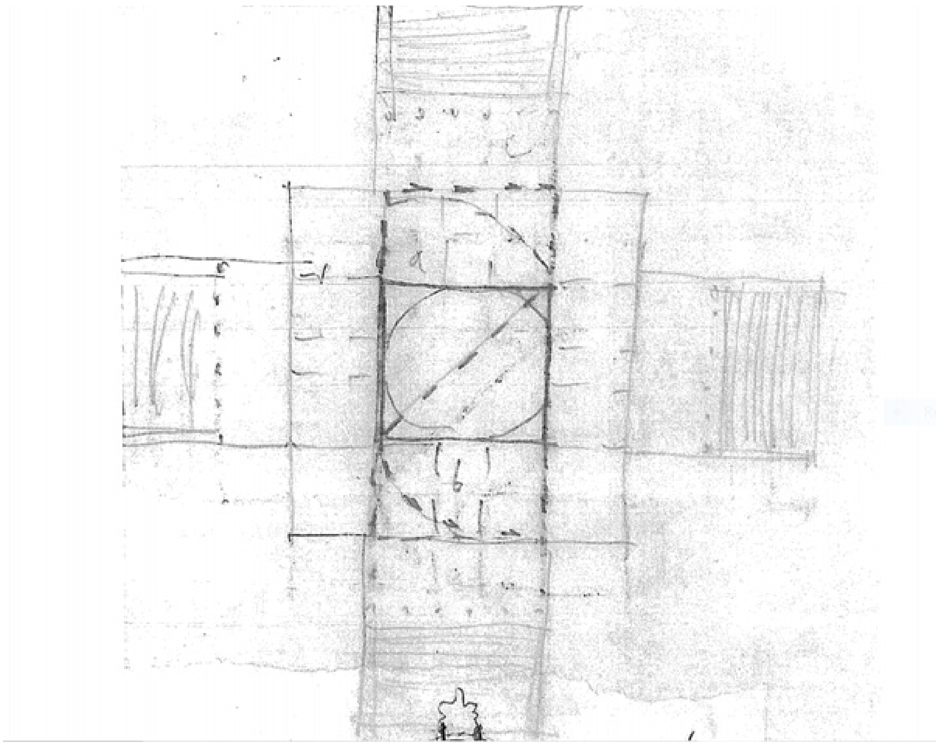


Figure Q: root two rectangles proportioning tool for the plan of the Rotunda, from a sketch by author ("Portrait of a bridge…", 2019)

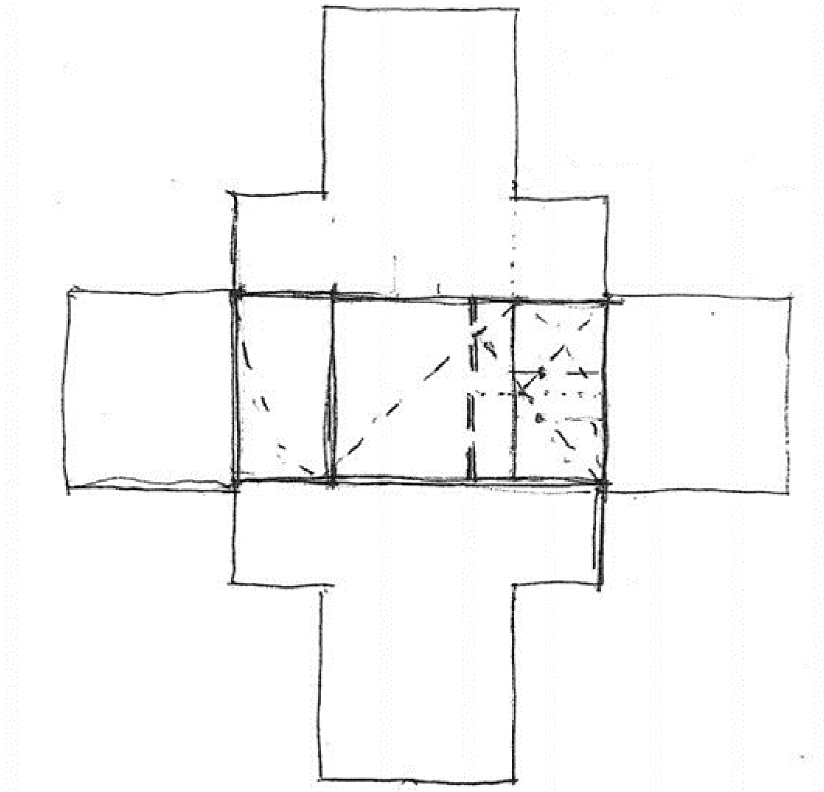


Figure C: plan of Rotunda with root two rectangles: proposed proportionality analysis and corridors based on that proportioning tool, from a sketch by author ("Portrait of a bridge…", 2019)

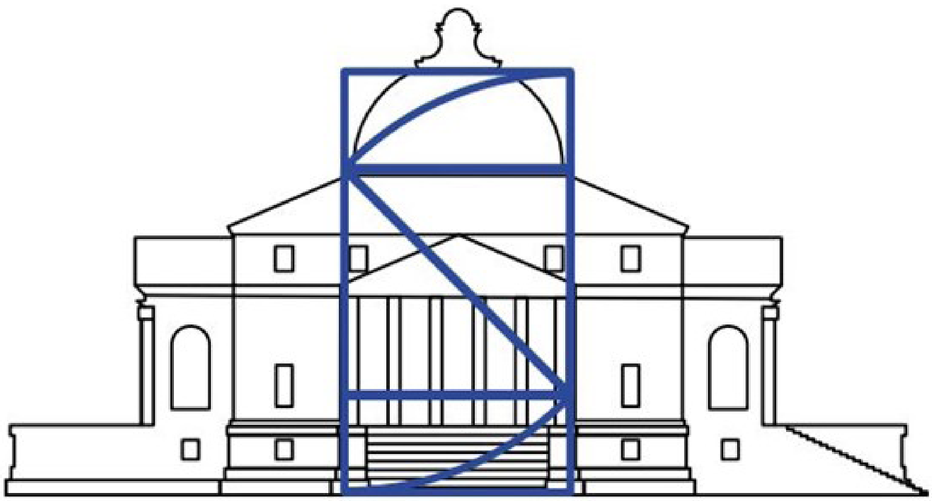


Figure E: elevation of Rotunda with root two rectangle proportioning tool: proposed proportionality analysis, from a sketch by author ("Portrait of a bridge…", 2019). It must be noted that Palladio's elevation had no lantern at the top of the dome, while Bertotti Scamozzi's drawing includes one.

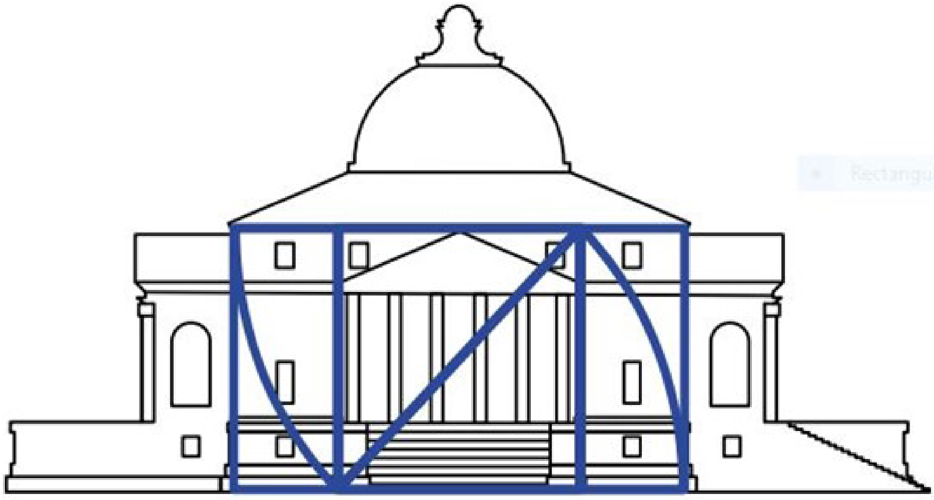


Figure F: elevation of Rotunda with root two rectangle proportioning tool: proposed proportionality analysis, from a sketch by author ("Portrait of a bridge…", 2019).

v. https://www.youtube.com/watch?v=GmZlAVRKx8I, accessed 16/09/2020.

A You Tube documentary[v] (whose authors also, like Serlio's translators Hart and Hicks, might not be aware of the prevailing/predominant use of the Root two rectangle during Renaissance), shows that the superimposition on the Rotunda Villa plan of Golden section Rectangles is also possible, which would somehow substantiate the hypothesis that a geometric construction with the Dürer -like first "geometric" method contains at the same time Root Two rectangles and Golden Section rectangles? It is noted that both Root Two rectangles and Golden Section rectangles can be conveniently divided into smaller rectangles that are proportional to the bigger ones, in line with Palladio's recommendations in his Four Books to have rooms with different, but "compatible", proportional, measurements - and heights when it comes to ceilings. As stated earlier, it is Wittkower's statement that only root two was a number of relevance in Renaissance architectural proportionality theory that leads us to the conclusion that the somehow "easier" to draft root two rectangles were the driving proportioning tool of the Renaissance-enlightened architect when he designed (in a centrifugal way) the Pantheon -like Rotunda Villa. Rachel Fletcher places the Golden Section in/on Villa Emo - a hypothesis cathegorically refuted by March.

The much cited "rigorous bi-axiality of Palladio's Villa Rotonda" (Anderson, 1994) – "diminished or destroyed in the English examples" (Anderson, 1994) - is in fact, according to the author of this paper, more of an equally-rigorous tri-axiality (in a UCS – Universal Coordinates System - along the x, y and z axes).

The Missing Link: Tetraktys-Based Pythagorean Examples of Proportions in Palladio's Four Books on Architecture

Palladio's "harmonic" proportions are consistently based on the Pythagorean Tetraktys and Lambda. We note, after March (1998), that the numbers provided in his example - to find the height of a vault with the so-called "harmonic" proportions method - are all from the Tetraktys and Pythagorean Lambda.

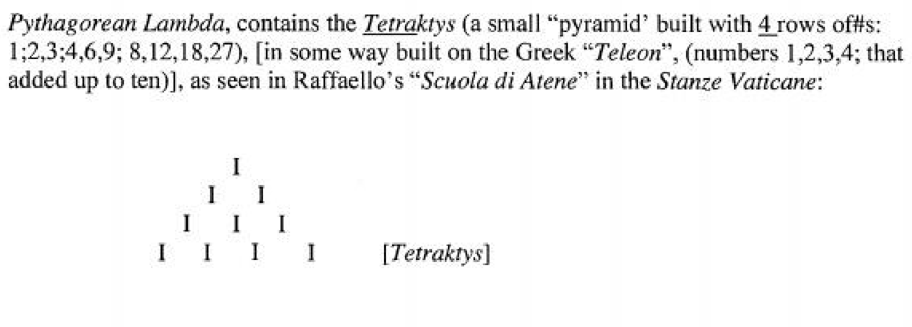

Pythagorean Lambda, contains the *Tetraktys* (a small "pyramid' built with 4 rows of#s: 1;2,3;4,6,9; 8,12,18,27), [in some way built on the Greek "*Teleon*", (numbers 1,2,3,4; that added up to ten)], as seen in Raffaello's "*Scuola di Atene*" in the *Stanze Vaticane*:

Figure L: Pythagorean Tetraktys (part of the Pythagorean Lambda) after author (2019), after March (1998)

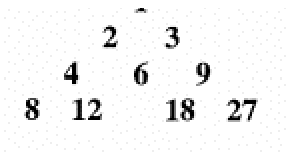


Figure T: the numbers of the Pythagorean "Tetraktys" (the Pythagorean Lambda truncated to the cubes), after March (1998)

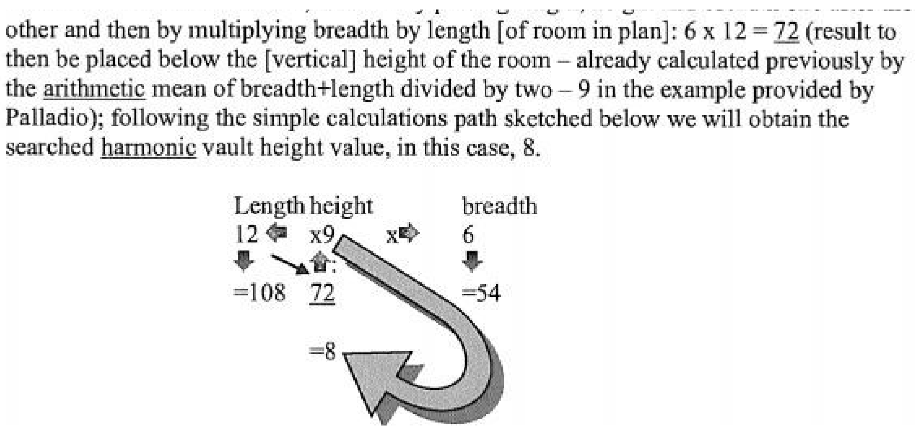

other and then by multiplying breadth by length [of room in plan]: 6 x 12 = 72 (result to then be placed below the [vertical] height of the room – already calculated previously by the arithmetic mean of breadth+length divided by two – 9 in the example provided by Palladio); following the simple calculations path sketched below we will obtain the searched harmonic vault height value, in this case, 8.

Figure H: method to calculate "harmonic" proportions to find height of vaults provided by Palladio in his Four Books on Architecture.

Also the previous methods mentioned by Palladio in his Four Books – arithmetic and first geometric method, let's say the one after Dürer that uses (in a quite mysterious way, given that to do it he uses a "bendable" ruler, the workings of which are not known exactly at present to the author) the golden mean to find the length of the side of the double of a cube – use numbers that are elements of the Tetraktys. 4,6 and 9, the Albertian "*proportione sesquialtera*", are all numbers of the Tetraktys and are aligned horizontally - the line being typical, according to March, 1998, of 'geometric' proportions in the Pythagorean Lambda. 6 and 12 are the numbers chosen for the arithmetic proportion example and they too, unsurprisingly, (even though only ever clearly stated by March, 1998) are from the "Tetraktys".

The Harmonic proportions can be visualised (after March, 1998) as small triangles made with three numbers of the Tetraktys.

To revise the "harmonic" proportions, let's draw the Tetraktys a bit like a 4x7 Matrix:

0	0	0	1	0	0	0
0	0	2	0	3	0	0
0	4	0	6	0	9	0
8	0	12	0	18	0	27

	Columns						
Rows	0	0	0	**1**	0	0	0
	0	0	**2**	0	**3**	0	0
	0	**4**	0	**6**	0	**9**	0
	8	0	**12**	0	**18**	0	**27**

Where the numbers n (n being the numbers identified by row & column) of Matrix 4x7 are:

	Columns						
Rows	n11	n12	n13	**n14**	n15	n16	n17
	n21	n22	**n23**	n24	**n25**	n26	n27
	n31	**n32**	n33	**n34**	n35	**n36**	n37
	n41	n42	**n43**	n44	**n45**	n46	**n47**

With n=number, first index = row, second index=column and, therefore:
1=n14; 2=n23; 3=n25; 4=n32; 6=n34; 9=n36; 8=n41; 12=n43; 18=n45; 27=n47[vi]

vi. Where:
n23=n14x2
n25=n14x3

n32=n23x2
n34=n23x3; n34=n25x2
n36=n25x3

n41=n32x2
n43=n32x3; n43=n34x2
n45=n34x3; n45=n36x2
n47=n36x3

We note the pattern that any number of inferior row is multiplied by two on its left to obtain numbers (double the "original" number) in rows that have higher indices, while any number of inferior row is multiplied by three on its right, obtaining numbers in rows that have higher indices that will be triple the "original" number (for instance refer to numbers 1,2 and 3 – etc.).

An "harmonic" proportion, based on the recommended method will be given by:

n34xn43/n34+n43/2=n41

In the example provided by Palladio:

6x12/6+12/2=

72/9=

=**8**

Similarly:

n36xn45/n36+n45/2=n43

9x18/9+18/2=

162/13.5=

=**12**

and so on, also in the Pythagorean Lambda where, for example:

12x24/12+24/2=

288/36/2=

288/18=

=**16**

vii. p. 98, MIT translation of Four Books on Architecture, 1997.

like the numbers used by Palladio for the design of the Palazzo, in Vicenza, of Mr Montano Barbarano, with Vicentine feet (in plan): **12, 16, 24**[vii].

viii. p.140, MIT translation, 1997.

ix. p. 138, MIT translation, 1997.

Both the Villa Thiene at Cicogna[viii] and the Villa Trissino at Meledo[ix] display the following Tetraktys' numbers: **36, 18, 12** (Vicentine feet, in plan), also in "harmonic proportion":

36x12/36+12/2=

432/48/2=

432/24=

=**18**

and that we can visualise as a small triangle in the Tetraktys as well.

Given the way the Tetraktys is built, we can find "arithmetic" means in it as follows:

an n14+n25/2=n23 pattern, which, substituting with the numbers is, for example:

1+3/2=4/2=2;

and an n41+n32/2=n34 pattern, with the numbers of the Tetraktys being another example:
8+4/2=12/2=**6**.

x. p.136, op.cit. (H&L, 1982).

The following statement by Howard and Longair is in contradiction with our hypothesis, as 5 is not a number of the Tetraktys, "However, it should be remembered that all the harmonic numbers are multiples of 2, 3, and 5."[x] 5 is not in the Tetraktys, so should it even be a so-called "harmonic number"? The topic should be researched more in depth.

The same authors state the following. "A clear majority of the main living rooms are given dimensions corresponding to Palladio's seven preferred room shapes. Yet as many as 39% of the principal rooms have different proportions from those he recommends. The fact that certain simple musical ratios are absent or poorly represented on the plans suggests that Palladio tended to use his own recommended ratios in preference to musical harmonic ratios. In conclusion we should consider the wider implications of these results. In its simplest terms our analysis shows that Palladio had a definite preference for multiples of 2, 3, and 5 in the

xi. (H&L, 1982) p.136

dimensions which appear in the plans in Book II of the Quattro libri. In interpreting these results, various hypotheses could be made, including: (a) that Palladio used a system of musical harmonies, as Wittkower suggests; or (b) that he adhered to his own simpler recommendations concerning room shapes; or (c) that he recognized the practical advantages of using simple, easily divisible numbers."[xi] (emphasis added).

xii. p.59 MIT translation, 1997.

All these hypotheses seem not [to be] completely satisfactory to the author– especially after March's work of 1998- a) old hypothesis b) most probably correct, although a partial explanation (a different hypothesis of the author is that, in three dimensions, a mix of the recommended "methods" might have been used for each building and that would have echoed in the plans – interrelated to the heights - in Ching's words: with "interlocked" proportions [Ching, 2007]) c) compatible with Tetraktys' theory, but not strong enough, for instance: no mention is made to the practical need Palladio always keeps in mind – and also openly states in the Four Books[xii] – to end up getting, for the different rooms, ceiling heights that can "work" "well" together "so we should make use of each of these heights depending on which one will turn out well to ensure that most of the rooms of different sizes have vaults of an equal height and those vaults will still be in proportion to them, so that they turn out to be beautiful to the eye and practical for the floor or pavement which will go above them because they will all end up on the same level", (p. 59 MIT, 1997) (emphasis added by author). Palladio's motives in that respect were extremely "practical", as he puts it, in nature - a massive distraction, if not an incredible "omissis"/omission, from the part of previous authors in general in not noticing, or downplaying, this motivational factor of the architect's work.

Conclusion

In terms of overall considerations about Palladio's work: he worked on an innovative modern methodology to design buildings, in order to provide "standardized" methods that architects could use repeatedly, with slight variations according to the different needs of each single project.

His proposed system was practical and linked sizes of rooms (length and breadth) to their heights, in a way that would make them most of the time the same height, in order to have the ceilings - and the upper floors - at the same level.

The method used by Palladio was not always the "harmonic" one (it could have been the arithmetic one and/or the geometric ones – like in the case of the Rotunda Villa), so it does not make sense to say that Palladio always used "harmonic" proportions, as it is said in Wittkower's theory of musical harmonic proportions and in other authoritative authors' hypotheses as well, such as, for instance, that of the extremely scrupulous and detail-oriented Howard and Longair.

The proportionality analysis of the Rotunda Villa turns out to always have been undertaken unsuccessfully in the past. This work has offered a different approach to "reading" the Rotunda Villa, aimed at unveiling its design proportionality patterns. It is our understanding that the Rotunda bears a tri-axial symmetry – along the coordinates x, y and z in space/ in 3D – and, although the sketches are approximate, the fact that also other designs of Palladio – see figures 1 and 2- are inscribable in Root two rectangles, would seem to corroborate this hypothesis. It would seem, furthermore, that the openings in the lateral corridors where placed there with the help of root two rectangles as proportioning tools (see figure C). Root two rectangles in plan (along both the x and y axes) and in elevation, along the z axis (see figures E and F), would seem to the author to have been used in the design phase. It remains ultimately uncertain which design and construction method Palladio was following while designing the Rotunda: his first "geometric" method, after Durer or simply the "doron" principle, after Barbaro, with the same dimensions "in every way" (March, 1998).

It would seem that, due to the omissions in the explanations of his recommended methods about how they actually worked/were "built", based on the Tetraktys and the Pythagorean Lambda, that he left mostly understated, many architects and authors have been, consistently, at a loss, so often missing the point of the work and labors of our very subtle, and at the same time extremely practical, Pythagorean Palladio.
Appendix: examples of "harmonic" proportion's numbers in the Tetraktys and in the Pythagorean Lambda.

1
2 3
4 6 9
8 12 18 27
16 24 36 54 81

Geometric proportion: 4,6,9 (a straight line, as noted by March, 1998, in the Tetraktys and in the Pythagorean "Lambda"/λ)

1
2 3
4 6 9
8 12 18 27
16 24 36 54 81

"Harmonic" proportion: 6,8, 12 (a small triangle, as already noted by March, 1998, in the Tetraktys)

1
2 3
4 6 9
8 12 18 27
16 24 36 54 81

"Harmonic" proportion: 9,12,18 (a small triangle, as already noted by March, 1998, in the Tetraktys)

				1				
			2		3			
		4		6		9		
	8		12		18		27	
16		24		36		54		81

12, 16,24: "Harmonic" proportion (in the Pythagorean "Lambda"/λ)

				1				
			2		3			
		4		6		9		
	8		12		18		27	
16		24		36		54		81

12,18,36: "Harmonic" proportion (in the Pythagorean "Lambda"/λ), as used by Palladio in Villa Thiene at Cicogna and at the Villa Trissino at Meledo.

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